

Application of Properties of Quadrilaterals in the Coordinate Plane

Objective

Students will be able to apply definitions and theorems to verify special quadrilaterals using coordinate geometry.

Core Learning Goals

2.1.2 The student will identify and/or verify properties of geometric figures using the coordinate plane and concepts from algebra.

Materials Needed

Worksheet, HSA Formula sheet, graph paper

Pre-Requisite Concepts Needed

Students should be able to apply the slope formula and be able to identify parallel or perpendicular lines after computing the slope. Students have been introduced to the properties of parallelograms. Usually the next section in geometry texts is proving a quadrilateral is a parallelogram and this lesson could serve as the next day's lesson after that introductory day.

Approximate Time

One to two 45-minute lessons depending on the level of student you are working with. This is an excellent way to have students practice BCR's-for all levels of students.

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Lesson plan

Warm-Up/Opening activity

Find the slope of the lines containing the following segments:

1. \overline{AB} given A (3, 4) and B (5, 8)
2. \overline{CD} given C (2, 1) and D (-4, 4)
3. \overline{EF} given E (2, 3) and F (2, 8)
4. \overline{GH} given G (3, 3) and H (7, 3)
5. \overline{IJ} given I (9, 10) and J (5, 2)

Have students apply the slope formula. To save time you could have 5 groups, or if you have 5 rows of students, have each student do one then compile the information.

Answers:

1. \overline{AB} : slope = 2
2. \overline{CD} : slope = $-\frac{1}{2}$
3. \overline{EF} : slope is undefined
4. \overline{GH} : slope is 0
5. \overline{IJ} : slope is 2

Discuss which segments are parallel (having the same slope) and which segments are perpendicular (having slopes that are negative reciprocals). Also included for discussion are pairs of segments that have zero slope (horizontal) as well as undefined slope (vertical).

Development of Ideas

Key to the lesson: Today we are going to apply the slope formula, the midpoint formula, and the distance formula to justify that a given quadrilateral is a parallelogram.

Worksheet: **Quadrilaterals in the Coordinate Plane**

Example 1: Given: A (4, 3) B (8, 3) C (9, 10) D (5, 10) are points of a quadrilateral. Is quadrilateral ABCD a parallelogram? Use mathematics to justify your answer. (graphing is not acceptable).

Method 1: *Let's show that this is a parallelogram using the definition of parallelogram. Ask: So what do we need to show? Both pairs of opposite sides are parallel. We might be able to see this better if we make a rough*

Application of Properties of Quadrilaterals in the Coordinate Plane

Development of Ideas (Continued)

Example 1 Method 1 (Continued)

sketch so we know which sides we are talking about. After making the sketch of ABCD (this may be a rough sketch with no orientation to the given coordinates), have students decide which pairs of sides need to be parallel. Then have them find the slopes. (You can save time by dividing up the labor.)

After computing the slopes, they should find that the slope of \overline{AB} and \overline{DC} is 0, and the slope of \overline{BC} and \overline{AD} is 7.

Justification of method one: *Now that we have the slopes computed we need to write a sentence or two explaining what we did and why we did it. I used the slope formula to find the slopes of the sides of the quadrilateral. The slope of $\overline{AB} = \overline{DC} = 0$ and the slope of $\overline{BC} = \overline{AD} = 7$. Both pairs of opposite sides have the same slope so I know that these sides are parallel. Therefore ABCD is a parallelogram by definition: a parallelogram is a quadrilateral with two pairs of parallel sides.*

Method 2: *What is another way to prove that ABCD is a parallelogram?* One of your students will respond: *Both pairs of opposite sides are congruent.* Then: *Which sides would we want to get congruent?* (Same sides that we used before.) *Which formula will we use this time?* (distance formula). *So, we need to show?* ($AB = DC$ and $BC = AD$). Have the students apply the distance formula. To save time, assign different groups the different lengths. After computing the lengths, they should have found that $AB = DC = 4$ and $BC = AD = \text{square root of } 50$, or approx. 7.07. Have the groups write their justification. Let them volunteer to share their work. *Justification: by applying the distance formula I found two pairs of congruent opposite sides:*

$$AB = \sqrt{0^2 + 4^2} = 4$$

$$DC = \sqrt{0^2 + 4^2} = 4$$

$$AD = \sqrt{7^2 + 1^2} = \sqrt{50}$$

$$BC = \sqrt{7^2 + 1^2} = \sqrt{50}$$

ABCD is a parallelogram by the theorem: If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

At this point, you may be nearing the end of class time and may give the following assignment: Verify ABCD is a parallelogram by choosing one pair of opposite sides and showing them parallel and congruent. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

Application of Properties of Quadrilaterals in the Coordinate Plane

Development of Ideas (Continued)

If you are not at the end of the period, you could verify ABCD is a parallelogram by showing that the diagonals bisect each other. (Show that both diagonals have the same midpoint, using the midpoint formula.)

Extensions

After investigating other quadrilaterals students could then: Verify parallelogram EFGH is a rectangle (or square, or rhombus). You could have groups of students use graph paper to plot a special quadrilateral then have them share the coordinates with another group. The second group could discover through application of properties the type of quadrilateral that is formed with the given coordinates. Students could show sides are perpendicular, using the slope formula or that the diagonals are congruent using the distance formula, or use other appropriate properties.

Some students may prefer using a two column proof to justify today's work.

Alternate assignment

If A(2,1) B(5,4) C(8,12) D(5,9) are the coordinates of a quadrilateral, verify that quadrilateral ABCD is a parallelogram. Use mathematics to justify your answer. (*Do the problem 4 different ways.)

1. Both pairs of opposite sides parallel.
2. Both pairs of opposite sides congruent.
3. One pair of sides congruent and parallel.
4. Diagonals bisect each other.

* Not HSA: In HSA we may ask students to prove quadrilateral ABCD is a parallelogram but only 1 method would be necessary.

Answers to alternate assignment

1. By using the slope formula, the slope of \overline{AB} and \overline{CD} is 1: the slope of \overline{AD} and \overline{BC} is $\frac{8}{3}$. Because the slopes of these pairs of sides are the same we can say ABCD is a parallelogram by definition: if both pairs of opposite sides are parallel then the quadrilateral is a parallelogram.
2. By using the distance formula, the length of \overline{AB} and \overline{CD} is $\sqrt{18}$ and the length of \overline{AD} and \overline{BC} is $\sqrt{73}$. Since both pairs of opposite sides are congruent we know ABCD is a parallelogram by the theorem: if both pairs of opposite sides are congruent then the quadrilateral is a parallelogram.

Application of Properties of Quadrilaterals in the Coordinate Plane

Development of Ideas (Continued)

Alternative Assignment Answers (Continued)

3. By using the slope formula, the slope of \overline{AB} and \overline{CD} is 1. The length of these same sides is $\sqrt{18}$. ABCD is a parallelogram by the theorem: if one pair of opposite sides is both parallel and congruent then the quadrilateral is a parallelogram.
4. By using the midpoint formula, the midpoints of diagonals \overline{AC} and \overline{BD} are the same: (5,6.5). ABCD is a parallelogram by the theorem: if the diagonals bisect each other then the quadrilateral is a parallelogram.

Application of Properties of Quadrilaterals in the Coordinate Plane

Quadrilaterals in the Coordinate Plane

Warm-Up

Find the slope of the lines containing the following segments:

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 2. \overline{CD} C (2, 1) and D (-4, 4)
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 4. \overline{GH} G (3, 3) and H (7, 3)
 5. \overline{IJ} I (9, 10) and J (5, 2)
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1. Given: A (4, 3), B (8, 3), C (9, 10), and D (5, 10) are vertices of a quadrilateral. Is quadrilateral ABCD a parallelogram? Use mathematics to justify your answer. (graphing is not acceptable).
 2. Verify ABCD is a parallelogram by choosing one pair of opposite sides and showing them parallel and congruent. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation

Alternate assignment

3. If A(2, 1), B(5, 4), C(8, 12), and D(5, 9) are the vertices of a quadrilateral, verify that quadrilateral ABCD is a parallelogram. Use mathematics to justify your answer. (Do the problem 4 different ways.)
 1. Both pairs of opposite sides parallel.
 2. Both pairs of opposite sides congruent.
 3. One pair of sides congruent and parallel.
 4. Diagonals bisect each other.